High School Algebra Playlist: Factoring Quadratic Expressions

Aligns with CCSS.Math.Content.HSA.SSE.B.3.a: Factor a quadratic expression to reveal the zeros of the function it defines.

Related Standards

- CCSS.Math.Content.HSA.SSE.A.2: Use the structure of an expression to identify ways to rewrite it.
  For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

- CCSS.Math.Content.HSA.REI.B.4: Solve quadratic equations in one variable.

Objectives

In this module, you will learn and practice the following skills:

- factor the quadratic expression within a quadratic equation

- identify the solutions of a quadratic equation from its factored form

Let's get started!

Key Terms

- A **factor** is a number that is multiplied by another number or by an expression to make a product. In the context of quadratic expressions, factors are binomials.

- A **quadratic equation** is a polynomial of degree 2, typically written $ax^2 + bx + c = 0$.

- A **binomial** is a polynomial with two terms, such as $5m^2 + 4m$. In the context of quadratic expressions, the binomials are of the form $ax + b$.

- The **FOIL** method is a mnemonic for multiplying two binomials. The product is the sum of multiplying the First, Outer, Inner, and Last terms in each binomial.

Connections

- https://openstaxcollege.org/textbooks/algebra-and-trigonometry; section 2.5

- https://openstaxcollege.org/textbooks/algebra-and-trigonometry; section 1.4

- https://openstaxcollege.org/textbooks/algebra-and-trigonometry; section 5.1

Factoring Quadratic Expressions

(CCSS.Math.Content.HSA.SSE.B.3.a)

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If your students…

Treat every quadratic expression as though a were 1:

Many students become adept at factoring quadratic equations of the form $x^2 + bx + c = 0$ but then continue to treat every equation as though the leading term has a coefficient of 1. Have them use FOIL to check their answers; they will see that the resultant product does not have the correct a value.

Treat every quadratic equation as though the right-hand side were 0:

Students typically do well solving equations such as $x^2 - 5x + 4 = 0$ by factoring, but then they focus only on the quadratic expression on the left-hand side, and they wind up mishandling an equation such as $x^2 - 5x + 4 = -2$ and mistakenly obtain the "solutions" $x = 1$ and $x = 4$. Have them substitute their solutions for the variable x into the original expression; if indeed $x = 1$ and $x = 4$, then those two values substituted for x should give -2.

Give the answer as $x = 0$:

Some students will factor a quadratic expression, getting something like:

$$x^2 - 5x + 6 = 0$$
$$(x - 3)(x - 2) = 0$$

and then tell you that $x = 0$. Remind them that the solution they are looking for is what makes, say, $x - 3$ equal to 0. Have them write it out as $x - 3 = 0$, as a separate equation, to see that they are looking for $x = 3$ as an answer. And have them graph the corresponding function $y = x^2 - 5x + 6$ to see the zeros.

Give the wrong sign for solutions:

Students often solve a quadratic equation of the form $x^2 - 5x + 6 = 0$ by thinking "I am looking for two numbers whose product is 6 and whose sum is -5"; then they give answers of $x = -2$ and $x = -3$. Other students correctly factor the equation as $(x - 3)(x - 2) = 0$ but then pick up the typographic "-2" and "-3" as the values for x. Again, remind them that they are looking for the value that makes, say, $x - 2$ equal to 0.