High School Functions Playlist: The Radian Measure of an Angle

Aligns with *CCSS.MATH.CONTENT.HSF.TF.A.1*:

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Related Standards

- *CCSS.MATH.CONTENT.HSF.TF.A.2*: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- *CCSS.MATH.CONTENT.3.NF.A.3*: Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the value of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their value for x, where x is any real number.
- *CCSS.MATH.CONTENT.3.NF.A.4*: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Introducing the Standard

Up to this point, when you have worked with angles, angle measures have been expressed in degrees. A degree is defined as the measure of an angle that turns through 1/360 of a circle. As a unit of measurement, the degree has worked just fine when solving geometry problems. When two lines are perpendicular to each other, they form angles of measure 90°. The measures of the interior angles of a triangle sum to 180°. The length of a circular arc subtending an angle *q*, measured in degrees, is found by multiplying the circumference $C = 2\pi r$ by $(\theta/360)$, where *r* is the radius of the circle:

$$s = 2\pi r\left(\frac{\theta}{360}\right) = \left(\frac{\pi}{180}\right)r\theta$$

As you begin to work with trigonometric functions, thinking of angle measures as numbers typically falling between 0° and 180° or, depending on the problem, maybe between 0° to 360°, will be somewhat limiting. The location of the end of a hand on an old-fashioned clock, for example, depends on the angle the hand makes with the horizontal. But since that angle increases linearly with time, which has no particular limit, thinking of the angle itself as having no particular limit becomes more helpful.

You will also find that working with trigonometric functions using degrees means that a factor of $(\pi/180)$ begins following you everywhere. This factor appears in the above equation for arc length. The area of a sector involves the same factor. It also happens to be the magnitude of the slope of the sine and cosine functions whenever they pass through zero if the angle is measured in degrees. There is also a useful approximation for small angles; $\sin\theta \cong \left(\frac{\pi}{180}\right)\theta$, assuming the angle is measured in degrees. Can you calculate 180 times the sine of 1°?

To broaden how we think about angles as we begin working with trigonometric functions in more depth, and to simplify some of the related calculations, we begin using a new unit of measure for angles, the **radian**.

High School Functions Mini-Module: The Radian Measure of an Angle

Aligns with *CCSS.MATH.CONTENT.HSF.TF.A.1:*

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Understand the radian measure of an angle as it relates to a unit circle.

One **radian** is defined as the measure of an angle where the arc subtended by the angle has a length equal to the radius. This means that the ratio of the arc length to the radius is the measure of the corresponding angle in radians. When a circle is a **unit circle** (radius equal to 1), the length of a 1-radian arc is 1.

If your students…

Do not understand the advantages of the radian measure of an angle, believing that the degree measure is sufficient:

WATCH:

https://www.opened.com/video/introduction-to-radians-khan-academy/180366

https://www.opened.com/video/example-radian-measure-and-arc-length/116367

Do not understand that a unit circle is a circle of radius 1, with all other circles being scaled versions of the unit circle:

WATCH:

https://learnzillion.com/lesson_plans/6517-learn-the-properties-of-the-unit-circle-in-terms-of-radians#fndtn-lesson

Do not understand the definition of a radian:

WATCH:

https://learnzillion.com/lesson_plans/5074-understand-radians-in-terms-of-the-properties-of-circles#fndtn-lesson

For extra practice, see the Assessment/Practice sections of:

https://learnzillion.com/lesson_plans/713-understand-radians-as-the-relationship-between-the-radius-and-circumference-in-a-circle-by-deriving-a-formula-for-radian-from-a-circle#fndtn-lesson

https://learnzillion.com/lesson_plans/710-understand-the-unit-circle-by-applying-knowledge-of-radians#fndtn-lesson