High School Algebra Playlist: Solving Quadratic Equations

Aligns with CCSS.Math.Content.HSA.REI.B.4.b: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Related Standards

- CCSS.Math.Content.HSA.SSE.B.3.a: Factor a quadratic expression to reveal the zeros of the function it defines.
- CCSS.Math.Content.HSA.SSE.B.3.b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- CCSS.Math.Content.HSA.REI.B.4.a: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- CCSS.Math.Content.HSN.CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.

Objectives

In this module, you will learn and practice the following skills:

- choose a method for solving a quadratic equation
- write complex solutions when appropriate

Let's get started!

Key Terms

- A **quadratic equation** is a polynomial of degree 2, typically written $ax^2+bx+c=0$.
- **Completing the square** is the process of converting an expression, such as a quadratic expression, into a perfect square by adding or subtracting terms on both sides.
- The **quadratic formula** is a formula that determines the roots of a quadratic equation from its coefficients.
- A **complex number** can be written as $a+bi$, where a and b are real and i is an imaginary number whose square equals -1.

Connections

- https://openstaxcollege.org/textbooks/algebra-and-trigonometry; section 2.4
- https://openstaxcollege.org/textbooks/algebra-and-trigonometry; section 2.5.4

Welcome

There are several ways to solve a **quadratic equation**, a polynomial equation of degree 2. You can choose your method of solution depending on the form of the equation.

As you solve for *x*, you may encounter a situation in which you are taking the square root of a negative number. You express the solution as a **complex number**.

What are the solutions of the quadratic equation $x^2+4=0$?

Watch!

For a quick overview of solving quadratic equations, watch these videos:

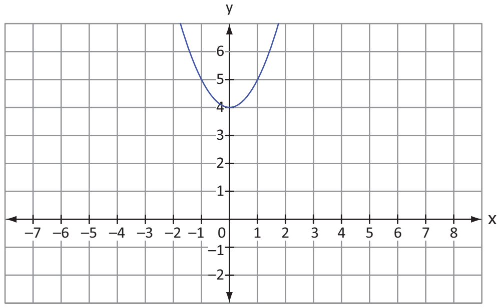
- https://www.opened.com/video/introduction-to-the-quadratic-equation/183649
- https://www.opened.com/video/completing-the-square-old-school/111607
- https://www.opened.com/video/example-problems-applying-the-quadratic-formula/183650
- https://www.opened.com/video/imaginary-roots-of-negative-numbers/181355

Focus: Complex Numbers

The quadratic formula includes taking the square root of a negative number. The solutions are not real numbers but exist in the complex plane; you express them as complex numbers.

Answers

Remember that finding the solutions to $x^2+4=0$ is like asking, "When does the graph of $y=x^2+4$ intersect the graph of $y=0$?" That is, where does the parabola cross the x-axis?



This graph never crosses the x-axis. There are no real values of x that make the equation "work".

However, there are imaginary values:

$$x^2+4=0$$
$$x^2=-4$$
$$x=\pm\sqrt{-2}$$
$$x=\pm 2i$$

Watch!

For a quick overview of complex numbers, watch these videos:

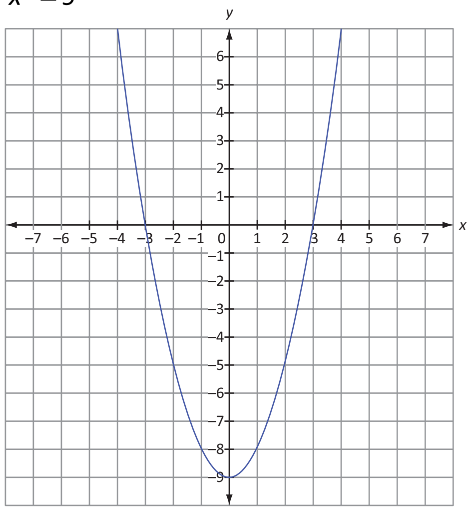
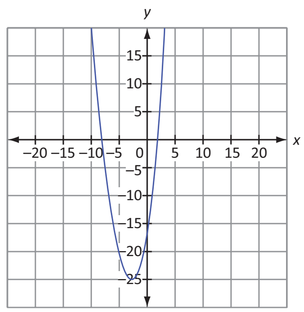
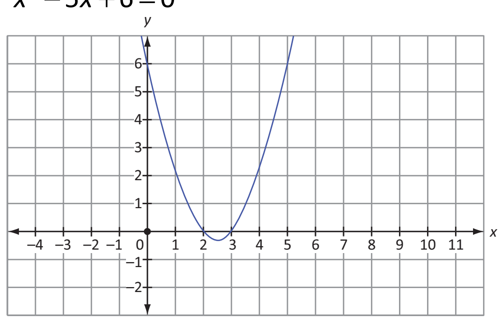
- https://www.opened.com/video/complex-number/3534952
- https://www.opened.com/video/using-complex-numbers-the-imaginary-number-i/1001395

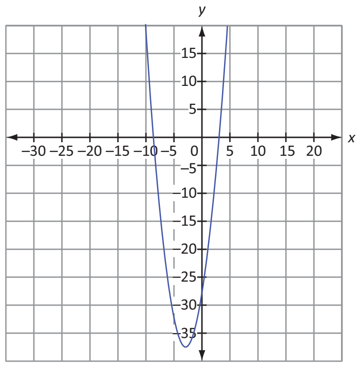
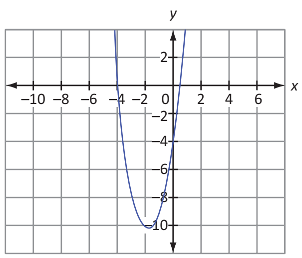
How can you tell, while solving a quadratic equation, how many real solutions it has?

Explore

There are two very different skills that will be covered in this module. The first is knowing what method to use to solve a quadratic equation. The second is representing complex solutions when they arise.

Very roughly speaking, the method you choose depends on how complicated the quadratic equation is. You can see if one of the more straightforward methods "works" easily before you move on to the next.

Method name or description	Example	Example Solution
Inspection: looking at the equation and doing minimal work.	$x^2 = 9$  	Take the square root of each side: $x^2 = 9$ $x = \pm\sqrt{9}$ $x = \pm 3$
Taking the square root. Useful in the case when the left side is a perfect square trinomial and the right side is a perfect square constant (including 0).	$x^2 + 6x + 9 = 25$  	Take the square root of each side and simplify: $x^2 + 6x + 9 = 25$ $(x+3)^2 = 25$ $x + 3 = \pm 5$ $x = 2; x = -8$
Factoring: finding the two binomials whose product is the quadratic expression. Factoring doesn't work easily if the roots are not integers or fractions.	$x^2 - 5x + 6 = 0$  	Try factors of the constant *c* until the binomials produce the sum *b*: $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2; x = 3$

Completing the square: converting an expression, such as a quadratic expression, into a perfect square by adding or subtracting terms on both sides. Completing the square can involve work with fractions.	$x^2+6x=27$ 	Recognize that x^2+6x looks like the start of $(x+3)^2$, so add the constant: $x^2+6x+9=27+9$ $(x+3)^2=27+9$ $(x+3)^2=36$ $x+3=\pm\sqrt{36}=\pm6$ $x+3=+6 \rightarrow x=6-3=3$ $x+3=-6 \rightarrow x=-6-3=-9$
Using the **quadratic formula**. This method always works, but you may do extra calculations that you can avoid by using simpler methods.	$2x^2+7x-4=0$ 	Use the values of a, b, and c: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\dfrac{-(7)\pm\sqrt{(7)^2-4(2)(-4)}}{2(2)}$ $x=\dfrac{-7\pm\sqrt{49+32}}{4}$ $x=\dfrac{-7\pm\sqrt{81}}{4}=\dfrac{-7\pm9}{4}$ $x=\dfrac{-7+9}{4}=\dfrac{1}{2}$; $x=\dfrac{-7-9}{4}=-4$

What happens when you have an equation such as $x^2+25=16$? It becomes $x^2=-9$, and $x=\pm\sqrt{-9}$. Remember that you can write this as $x=\pm3i$. The square root of 9 "comes out" of the radical as a 3, leaving i. Written as complex numbers with a real part of 0, the solutions are $0+3i$ and $0-3i$, or, more simply, $3i$ and $-3i$.

Similarly, as you use the quadratic formula to solve $x^2-8x+18=0$, you wind up with these complex solutions, with $a=1; b=-8; c=18$:

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
x &= \frac{-(-8) \pm \sqrt{(-8)^2-4(1)(18)}}{2(1)} \\
x &= \frac{8 \pm \sqrt{64-72}}{2} \\
x &= \frac{8 \pm \sqrt{-8}}{2} \\
x &= \frac{8 \pm 2i\sqrt{2}}{2} \\
x &= 4 \pm i\sqrt{2}
\end{aligned}
$$

There are two complex solutions, $4-i\sqrt{2}$ and $4+i\sqrt{2}$.

You get solutions that are complex only when the expression under the radical sign is negative – that is, when b^2-4ac is negative. This expression is called the *discriminant*. You can calculate it directly from the coefficients in order to determine whether the corresponding quadratic function intersects the *x*-axis twice, once, or zero times. (Remember to rewrite the equation as $ax^2+bx+c=0$, and remember that *b* and *c* may be 0.)

Value of discriminant b^2-4ac	Number of intersections with *x*-axis	Number and type of solutions
positive	2	Two real solutions
zero	1	One real solution
negative	0	Two complex solutions

Watch!

For more information about solving quadratic equations, watch these videos:

- https://www.opened.com/video/how-to-use-the-quadratic-formula/116351
- https://www.opened.com/video/solve-a-quadratic-equation-by-taking-a-square-root/139782

Practice!

You can practice solving quadratic equations by completing this activity:

- https://www.opened.com/homework/n-cn-7-solve-quadratic-equations-with-real-coefficients/3692508